Math 1, Lecture 3
Precalculus

Midterm 2

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books, notes, or calculators. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: Solutions

UID: _____

Section: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

You throw a ball into the air on planet Zzykon*. Its height in meters above the ground as a function of time is $h(t) = 14 + 12t - 2t^2$.

(a) [2pts.] From what height was the ball thrown?

(b) [4pts.] What is the maximum height the ball reaches?

(c) [4pts.] At what time does the ball hit the ground?

*Fictional planet used because acceleration due to gravity is a less convenient number on Earth.

(a) $h(0) = 14$ m

(b) Maximum occurs at vertex

$$t = \frac{-b}{2a} = \frac{-12}{-4} = 3 \text{ s}$$

$$h(3) = 14 + 36 - 18 = 32 \text{ m}$$

(c) Want $0 = 14 + 12t - 2t^2$

$$0 = -2(t^2 - 6t - 7)$$

$$0 = -2(t - 7)(t + 1)$$

$\boxed{t = 7}$ $t = -1$ ✗

Problem 2.

Consider the rational function

$$f(x) = \frac{2x^2 - x - 3}{(x+2)(x-4)}$$

(a) [5pts.] Find all asymptotes and intercepts of $f(x)$.

(b) [5pts.] Draw a graph of $f(x)$.

(a) $f(x) = \dfrac{(2x-3)(x+1)}{(x+2)(x-4)}$

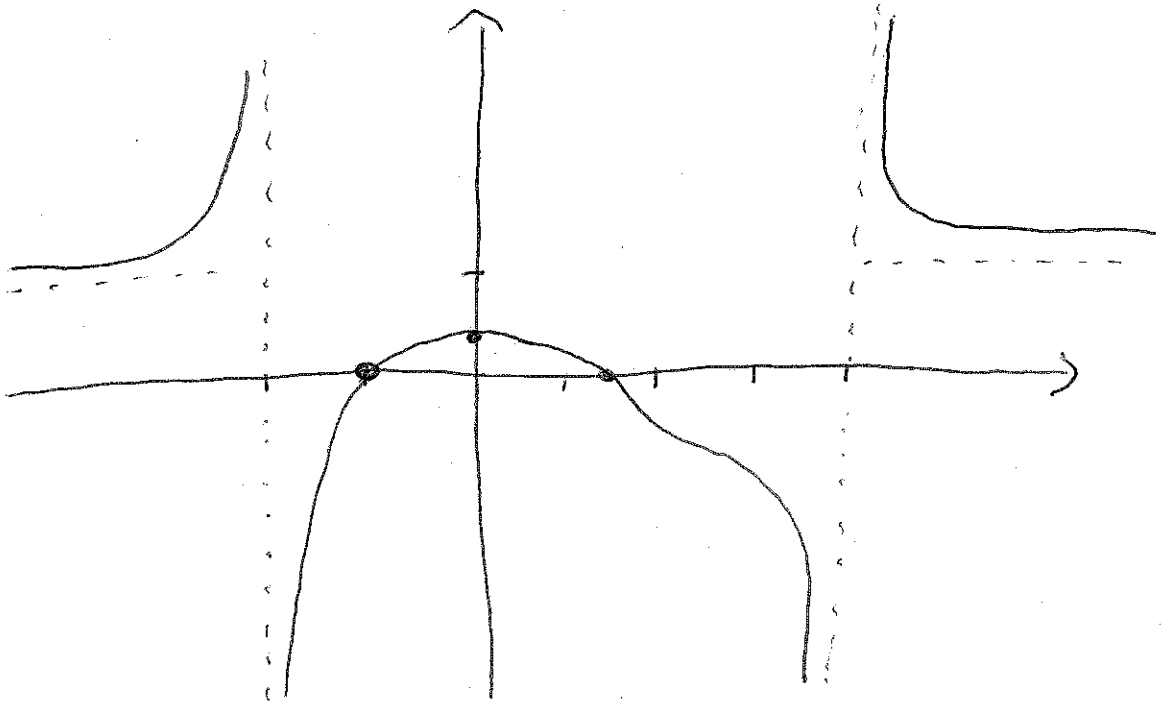
Vertical asymptotes $x = -2$ $x = 4$

Horizontal Intercepts $x = \frac{3}{2}$ $x = -1$

Vertical Intercept $y = f(0) = \frac{3}{8}$

Horizontal asymptote $y = \frac{2x^2}{x^2} = 2$

(b)



Problem 3.

Consider the polynomial $f(x) = (1-x)(x-6)^2(4x+8)$.

(a) [5pts.] Describe, using the notation introduced in class, the long-term behavior of $f(x)$.

(b) [5pts.] Solve the polynomial inequality $f(x) < 0$.

(a) Leading term is $-x(x^2)(4x) = -4x^4$

As $x \to \infty$, $F(x) \to -\infty$.

As $x \to -\infty$, $F(x) \to -\infty$.

(b) Horizontal intercepts at $x = -2, 1, 6$.

	$(-\infty, -2)$	$(-2, 1)$	$(1, 6)$	$(6, \infty)$
$1-x$	+	+	−	−
$(x-6)^2$	+	+	+	+
$4x+8$	−	+	+	+
Total	−	+	−	−

Solution $(-\infty, 2) \cup (1, 6) \cup (6, \infty)$

Problem 4.

A population of deer in a forest was 30 in 2010 and is now 120.

(a) [3pts.] Find an exponential model for the growth of the deer population.

(b) [3pts.] How many deer do you expect there to be in 2016?

(c) [3pts.] Assuming the population continues to expand at the same rate, at what time will the population reach 210 deer? Leave your answer in unsimplifed form.

(d) [1pts.] Will your model work indefinitely? Explain.

(a) $(0, 30)$ $\quad P(t) = 30\,b^t$

$(4, 120)$ $\quad 120 = 30\,b^4$

$4 = b^4$

$\sqrt{2} = b$

$\boxed{P(t) = 30(\sqrt{2})^t \text{ deer}}$

(b) $P(6) = 30(\sqrt{2})^6 = 30(8) = 240$ deer

(c) $210 = 30(\sqrt{2})^t$

$7 = 2^{t/2}$

$\log_2 7 = \frac{t}{2}$

$2\log_2 7 = t$ $\quad \left[\text{or } t = \frac{2\ln 7}{\ln 2} = \frac{2\log 7}{\log 2}\right]$

(d) No, eventually the deer will run out of resources or space.

Problem 5.

For each equation below, either solve for the variable or explain why this is impossible.

(a) [5pts.]

$$\ln(x-2)+\ln(x+3)=2\ln(x-1)$$

(b) [5pts.]

$$\frac{2^x-1}{2^x+2}=3$$

(a) $\ln((x-2))+\ln((x+3))=2\ln(x-1)$

$\ln((x-2)(x+3))=\ln((x-1)^2)$

$(x-2)(x+3)=(x-1)^2$

$x^2+x-6=x^2-2x+1$

$3x=7$

$x=\frac{7}{3}$

(b) $\frac{2^x-1}{2^x+2}=3$

$2^x-1=3(2^x)+6$

$-7=2(2^x)$

$\frac{-7}{2}=2^x$ Not possible, $2^x>0$. So no solutions.